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How Conservative Does the Central Banker Have to Be? On the Treatment of Expectations under Discretionary Policymaking

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ABSTRACT: This paper explores an issue that arises in the delegation process. The paper shows that a myopic central banker, one who treats expectations as constant in setting discretionary policy, can replicate the behavior of output and inflation under policy from a timeless perspective. For that to happen, society must delegate a price level target or a speed limit policy to a central banker who is more *weight-conservative* than society.

JEL Categories: E3, E5

Keywords: New Keynesian Model, Price Level Targeting, Speed Limit Policy, Conservative Central Banker

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I. INTRODUCTION

The rise of the New Keynesian or forward-looking model as the workhorse model to analyze monetary policy issues has generated renewed interest in the delegation issue. The need to vest a central banker with the authority to conduct monetary policy by discretion arises because society cannot tie the hands of the policymaker. Optimal pre-commitment is not possible. Policymaking by discretion is the only viable alternative. However, as pointed out by Woodford (1999), *pure* discretion turns out to be an inferior strategy as it does not introduce inertia into the conduct of monetary policy or provide a mechanism to influence forward-looking expectations. If the policymaker uses discretion in minimizing society's loss function, he counters a one-off positive cost-push shock by raising the interest rate to produce a negative output gap which in turn causes inflation to fall. Optimal policy by discretion does not directly affect the forward-looking expectations of inflation and causes the endogenous variables to adjust within the current period. In addition, the conduct of policy by discretion produces a stabilization bias, i.e. insufficient adjustment in the output gap relative to policy from a timeless perspective.

Society can introduce desirable inertia into the conduct of monetary policy and make the formation of forward-looking expectations of inflation susceptible to current policy action by delegating a specific loss function to a central banker, who acts with discretion. This delegated loss function is different from society's in that it features a different target variable. Thus, society instructs the policymaker to pursue an objective that does not appear in its own welfare function. Vestin (2000), Jensen (2002), and Walsh (2003) examine such delegated loss functions in the pure forward-looking model or in an extended version of it. Discretionary price-level targeting, nominal income growth targeting, or a speed limit policy introduces a lagged endogenous variable into the model. This has the effect of rendering policymaking inertial and making forward-looking expectations react to current policy in a way similar to policy from a timeless perspective. This channel through which the expectations of the public can be affected is of critical importance in the conduct of optimal discretionary policy. In the face of a positive cost-push shock, the policymaker can reduce both the rate of inflation and expected future inflation simultaneously. Reductions in the latter are achieved through keeping the output gap negative for a prolonged period of time.[1]

This paper analyzes the merits of discretionary price-level targeting and a discretionary speed limit policy in the pure forward-looking model. Our analysis focuses on the case where policy is determined by a myopic policymaker. A myopic policymaker treats the current expectation of the price level or inflation next period as constant when setting policy. Treating the expectation as constant has the effect of suppressing the link that exists between current policy action and the forward-looking expectations. Such myopic behavior on the part of the policymaker turns out to be of central importance in a setting where society delegates the conduct of monetary policy to a central banker who acts with discretion. The analytical results indicate that when the forward-looking expectations are treated as constant when policy is being set, then both discretionary price-level targeting and a discretionary speed limit policy can replicate the behavior of the output gap and the rate of inflation that eventuate under policy from a timeless perspective. The time series property of the cost-push shock has no bearing on this result. Specifically, society must delegate a price level target to a central banker who has greater aversion to price level variability than society has to inflation variability. In the alternative scenario of a speed limit policy, society must appoint a central banker who has a greater aversion to inflation variability than society. Both outcomes are interpreted as warranting the appointment of a conservative central banker.

The results reported in this paper differ markedly from those of Vestin (2000) and Walsh (2003). Vestin concludes that discretionary price level targeting can replicate the behavior of the output gap and inflation under policy from a timeless perspective only if the cost-push shock is a white noise process. He also reports that there is no clear-cut answer to the delegation issue under discretionary price level targeting. Walsh finds that a discretionary speed limit policy is always inferior to optimal policy from a timeless perspective, irrespective of the type of central banker appointed. The difference in results is attributable to the way the policymaker treats the forward-looking expectations at the policy-setting stage. Both Vestin and Walsh assume that the policymaker does not take the forward-looking expectations as given. Rather, the policymaker views the forward-looking expectations of the target variables as dynamic processes that respond to current policy action.

The remainder of the paper is organized as follows. Section II presents a simple variant of the forward-looking model and discusses optimal monetary policy from a timeless perspective. This form of optimal policy serves as the benchmark case against which discretionary price level targeting and a discretionary speed limit policy are compared. Section III analyzes price-level targeting under discretion while Section IV elaborates on a speed limit policy under discretion. The delegation issue under constant expectations is taken up in Section V. Section VI concludes.

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[1] Average inflation targeting (Nessén and Vestin (2005)) is another policy strategy that affects the formation of forward-looking expectations and introduces inertia into the conduct of policy.

II. THE MODEL AND OPTIMAL POLICY FROM A TIMELESS PERSPECTIVE

The Forward-Looking Model

The simple forward-looking model consists of the following two equations:

$$y_t = E_t y_{t+1} - a_1 r_t + v_t \qquad 1)$$

$$\pi_t = \beta E_t \pi_{t+1} + a y_t + u_t \qquad 2)$$

where

y_t is the output gap

r_t is the real rate of interest and

π_t is the rate of inflation.

u_t and v_t are autoregressive disturbances: $v_t = \gamma v_{t-1} + \hat{v}_t \qquad 0 \le \gamma < 1$

$$u_t = \rho u_{t-1} + \hat{u}_t \qquad 0 \le \rho < 1$$

Both a and a_1 are strictly positive parameters. β is the discount factor.

Equation 1) is the forward-looking IS curve. The current output gap depends on the current expectation of the output gap next period and is inversely related to the real rate of interest which serves as the policy instrument. Equation 1) assumes that the policymaker has full control over the setting of the real rate of interest.[2] Equation 2) represents the forward-looking Phillips Curve. The current rate of inflation depends on expected inflation next period and the current output gap.

Policy Objectives and Optimal Policy from a Timeless Perspective

Society is concerned about the variability of the rate of inflation and the output gap. Society's objectives appear in an intertemporal loss function that the policymaker strives to minimize at a given point in time. The policy objective takes the following form:

$$\underset{\pi,y}{Min} \; E_t \sum_{j=0}^{\infty} \beta^j \left(y_{t+j}^2 + \mu \pi_{t+j}^2 \right) \qquad 0 < \beta \le 1 \qquad 3)$$

[2] Suppressing the distinction between the nominal and real interest rate simplifies the algebraic analysis. Making this simplifying assumption has no material bearing on the results. Indeed, full control over the real rate of interest is necessary to implement a given policy successfully – be it under commitment or discretion.

The target variables are the rate of inflation and the output gap. The respective target value for the output gap and the rate of inflation is zero. The policymaker minimizes the above loss function with respect to the target variables subject to the constraint imposed by the Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + ay_t + u_t \qquad 4)$$

Hence the Lagrangean for the policy problem becomes:

$$\Gamma_t = E_t\{\sum_{j=0}^{\infty}\beta^j[(y_{t+j}^2 + \mu\pi_{t+j}^2) + \lambda_{t+j}(\beta\pi_{t+j+1} + ay_{t+j} + u_{t+j} - \pi_{t+j})\} \qquad 5)$$

Taking the derivative of Γ_1 with respect to the target variables ($y_1, \pi_1, y_2, \pi_2, y_3, \pi_3, \ldots$) yields the following first-order conditions under commitment:

$$2y_t + \lambda_t a = 0 \qquad \text{for periods } t = 1,2,3\ldots\ldots \qquad 6a)$$

$$2\mu\pi_t + \lambda_{t-1} - \lambda_t = 0 \qquad \text{for periods } t = 2,3\ldots\ldots \qquad 6b)$$

$$2\mu\pi_1 - \lambda_1 = 0 \qquad \text{for period } 1 \qquad 6c)$$

In 6a)-6c) $\beta = 1$.[3] What is remarkable about the first-order conditions is that the policymaker follows two different decision rules for setting the rate of inflation. In the initial, i.e. *start-up* period, he uses equation 6c) while for all subsequent periods he uses 6b) to set the rate of inflation. Inspection of 6b) and 6c) reveals that the optimizing condition for inflation in the initial period does not take account of expectations regarding future inflation while the optimizing condition in later periods does. Setting the rate of inflation according to 6c) is clearly suboptimal in the forward-looking model as *current* private sector expectations of inflation can be influenced by monetary policy. In sharp contrast, the policymaker employs 6a) in every period to set the output gap.

[3] Making this assumption allows for a straightforward interpretation of the intertemporal loss function in terms of the unconditional expectation of the loss function as explained at the end of the current section.

The existence of two distinctly different decision rules that governs the behavior of the rate of inflation gives rise to the time-inconsistency problem in the conduct of optimal monetary policy as aptly described by McCallum and Nelson (2004).

Policy from a timeless perspective circumvents the time-inconsistency problem. The timeless perspective assumes a stable macroeconomic environment where price stability has been achieved and inflationary expectations are well-anchored. The conduct of monetary policy is transparent, stable, and well-understood by the public. As a result, the condition that characterized the behavior of the rate of inflation in the *initial* period can be ignored.[4]
For periods $t=2, 3, \ldots$ combining equations 6a) and 6b) yields:

$$2\mu\pi_t - \frac{2y_{t-1}}{a} + \frac{2y_t}{a} = 0$$

$$\frac{1}{\mu a}(y_t - y_{t-1}) + \pi_t = 0$$

$$\theta(y_t - y_{t-1}) + \pi_t = 0 \qquad \theta = \frac{1}{\mu a} \tag{7}$$

Equation 7) represents the systematic *linear* relationship between inflation and the change in the output gap that characterizes the conduct of optimal monetary policy from a timeless perspective. The presence of the lagged output gap in the policy rule affords the policymaker to condition private sector expectations. It also makes current policy depend on the past behavior of the output gap. An important property of optimal monetary policy under policy from a timeless perspective is that the weight on the change in the output gap in the policy rule does not depend on the degree of persistence in the cost-push shock. The history dependence of monetary policy is thus not due to the nature of the cost-push shock but to the inherent inertial character of optimal policy.

To determine the behavior of the output gap and the rate of inflation under optimal policy from a timeless perspective, we combine the building blocks of the forward-looking model with the policy rule governing optimal policy. Substitute the Phillips Curve and the IS relation into equation 7) and solve the resulting expression for r_t.[5] This expression is then substituted back into the IS relation to obtain:

[4] See Woodford (1999) for an elegant description of the conditions necessary to implement monetary policy from a timeless perspective.
[5] For the sake of brevity, the policymaker's reaction function is not reported.

$$y_t = \frac{\theta}{\theta + a} y_{t-1} - \frac{1}{\theta + a}(E_t \pi_{t+1} + u_t) \qquad 8)$$

Combining equation 8) with equation 7) yields the reduced form equation for the rate of inflation:

$$\pi_t = \frac{a\theta}{\theta + a} y_{t-1} + \frac{\theta}{\theta + a}(E_t \pi_{t+1} + u_t) \qquad 9)$$

The expectation of the rate of inflation in period *t+1* is determined by applying the minimum state variable approach suggested by McCallum (1983). Due to the inertial character of optimal policy from the timeless perspective, the lagged output gap appears in the putative solution for the endogenous variables of the model:

$$y_t = \phi_{11} u_t + \phi_{12} y_{t-1} \qquad 10)$$

$$\pi_t = \phi_{21} u_t + \phi_{22} y_{t-1} \qquad 11)$$

After applying the method of undetermined coefficients, we obtain the following solutions for the output gap and the rate of inflation under optimal policy from a timeless perspective:[6]

$$y_t = \frac{1}{\tau} y_{t-1} - \frac{\mu a}{\tau - \rho} u_t \qquad 12)$$

$$\pi_t = \frac{(\tau - 1)}{\tau \mu a} y_{t-1} + \frac{1}{\tau - \rho} u_t \qquad 13)$$

$$u_t = \rho u_{t-1} + \hat{u}_t$$

$$0 \le \rho < 1 \qquad \tau = 1 + \frac{a^2 \mu}{2} + \frac{a\mu\sqrt{a^2 + \frac{4}{\mu}}}{2} > 1$$

Before proceeding, a word on the measurement of performance of a given policy strategy under certainty equivalence is in order. A natural measure of the variability of a target variable (around its mean) is its unconditional variance. Straightforward manipulation, which requires multiplying the intertemporal loss function by $1 - \beta$ and taking the limit as β approaches unity

[6]McCallum and Nelson (2004) adopt a similar solution procedure. Their algebraic analysis considers, however, only white noise disturbances.

turns the intertemporal loss function, which is expressed in terms of expected squared deviations of the target variables (around their target values) at $t=t_0$, into a linear combination of the unconditional variances of the target variables. More formally,

$$\lim_{\beta\to 1}(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^{(t-t_0)}[y_t^2+\mu\pi_t^2]. \qquad 14)$$

Evaluating this expression leads to a simple metric which consists of the weighted sum of the unconditional variances of the output gap and the rate of inflation:[7]

$$E(L_t)=V(y_t)+\mu V(\pi_t) \qquad 15)$$

Equation 15) is the performance measure of policy from society's point of view. To calculate expected losses under policy from a timeless perspective, we need to determine the variances of the output gap and the rate of inflation associated with equations 12) and 13). Both variances appear in the table below.

Table 1: The Variances of the Output Gap and the Rate of Inflation: The Timeless Perspective

$V(y_t)=$	$\dfrac{(\tau\mu a)^2(\tau+\rho)}{(\tau^2-1)(\tau-\rho)^3}\sigma_u^2$
$V(\pi_t)=$	$\dfrac{2\tau^2(1-\rho)}{(1+\tau)(\tau-\rho)^3}\sigma_u^2$

Note: $\sigma_u^2=\dfrac{1}{1-\rho^2}\sigma_{\hat{u}}^2$

Optimal policy from a timeless perspective is the benchmark case and forms the basis of the comparison with flexible price level targeting under discretion in Section III and a speed limit policy under discretion in Section IV.

III. FLEXIBLE PRICE LEVEL TARGETING UNDER DISCRETION

This section analyzes the case of discretionary flexible price level targeting. Society delegates a price level target to a myopic central banker who acts with discretion in the conduct of policy.

[7]Chapter 7 of Froyen and Guender (2007) provides further details on the individual steps of this transformation.

Under flexible price level targeting, the target variables are the price level and the output gap. The target for the price level is assumed constant through time, and the target for the level of real output is its potential level The expected loss function that the policymaker minimizes consists of the variance of the output gap and the variance of the price level (p_t):[8]

$$E[L_t] = V(y_t) + \hat{\mu}V(p_t) \qquad 16)$$

$\hat{\mu}$ indicates the extent to which the policymaker cares about the variability of the price level relative to the variability of the output gap.

In the current period, the policymaker decides on the systematic relationship between the target variables, the output gap and the price level. The parameter $\hat{\theta}$ represents the weight the policymaker attaches to the output gap relative to the deviation of the observed price level from its target level p^* in the policy rule.[9]

$$\hat{\theta}y_t + p_t - p^* = 0 \qquad 17)$$

To solve the model, we begin by rewriting equation 2) in terms of the price level and setting $\beta = 1$:[10]

$$p_t = \frac{E_t p_{t+1} + p_{t-1} + ay_t + u_t}{2} \qquad 18)$$

Next, substitute equation 18) and equation 1) into the policy rule, equation 17), and solve for the policy instrument r_t:

$$r_t = \frac{1}{a_1}(E_t y_{t+1} + v_t) - \frac{1}{a_1(2\hat{\theta} + a)}(p^* - E_t p_{t+1} + p^* - p_{t-1} - u_t) \qquad 19)$$

Equation 19) represents the reaction function of the policymaker. The setting of the instrument responds to the demand-side and the cost-push disturbances, the expected output gap next period,

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[8] The same transformation as in the previous section is applied to the intertemporal loss function which now includes the price level as one of the two target variables.
[9] The linear relationship between the price level and the output gap in equation 17) can be formally derived by taking an intertemporal perspective. See Guender (2006) for further details. Intuitively, the existence of a quadratic loss function gives rise to a linear policy rule.
[10] Initially, we consider the case where the cost-push shock is a white noise disturbance. The case of persistence in the cost-push shock is taken up later.

and the deviations of the lagged price level and the expected price level next period from the price level target.

To get the reduced form equation for real output, insert equation 19) into the IS relation:

$$y_t = \frac{2p^*}{2\hat{\theta}+a} - \frac{(E_t p_{t+1} + p_{t-1} + u_t)}{2\hat{\theta}+a} \qquad 20)$$

To recover the reduced form equation for the price level, combine equation 20) with the policy rule, equation 17):

$$p_t = \frac{ap^*}{2\hat{\theta}+a} + \frac{\hat{\theta}(E_t p_{t+1} + p_{t-1} + u_t)}{2\hat{\theta}+a} \qquad 21)$$

Under discretion, the policymaker acts myopically and takes the expectation of the price level next period as given. In setting policy, the policymaker thus ignores a dynamic aspect of the expectations formation process, the connection between $E_t p_{t+1}$ and p_t that exists under price level targeting. More specifically, treating the expectation of the price level next period as a constant suppresses the effect of a change in the setting of the policy instrument, which affects the current price level contemporaneously, on the expected price level next period. With $E_t p_{t+1}$ being treated as constant in equations 20) and 21), the variances of the output gap and the price level are equal to:

$$V(y_t^{PLT}) = \frac{1}{(a+2\hat{\theta})^2 - \hat{\theta}^2}\sigma_u^2 \qquad 22)$$

$$V(p_t^{PLT}) = \frac{\hat{\theta}^2}{(a+2\hat{\theta})^2 - \hat{\theta}^2}\sigma_u^2 \qquad 23)$$

The policymaker's objective can then be stated as:

$$\underset{\hat{\theta}}{Min}\ E[L_t] = V(y_t^{PLT}) + \hat{\mu}V(p_t^{PLT}) \qquad 16')$$

$$= \frac{1}{(a+2\hat{\theta})^2 - \hat{\theta}^2}(1+\hat{\mu}\hat{\theta}^2)\sigma_u^2$$

The optimal value of $\hat{\theta}$ which minimizes the policymaker's losses is given by:[11]

$$\hat{\theta} = \frac{3 - a^2\hat{\mu} + \sqrt{(-3 + a^2\hat{\mu})^2 + (4a)^2\hat{\mu}}}{4a\hat{\mu}} \qquad 24)$$

Recall that the policymaker takes the current expectation of the price level next period as given when determining the optimal value of the policy parameter. The actual behavior of the endogenous variables p_t and y_t is, however, influenced by $E_t p_{t+1}$ as shown by equations 20) and 21). Hence it is necessary to show how the expectation is formed. Let the putative solution for p_t be given by:

$$p_t = \phi_{20} + \phi_{21}u_t + \phi_{22}p_{t-1} \qquad u_t \sim N(0, \sigma_u^2) \qquad 25)$$

Then it follows that

$$E_t p_{t+1} = \phi_{20}(1 + \phi_{22}) + \phi_{21}\phi_{22}u_t + \phi_{22}^2 p_{t-1}. \qquad 26)$$

Inserting the above expressions for the price level and for the expectation of the price level into equation 21) and matching coefficients yields the following reduced form equation for the price level under discretionary policymaking:

$$p_t = \frac{2ap^*}{a + \Lambda} + \frac{2\hat{\theta}(u_t + p_{t-1})}{a + 2\hat{\theta} + \Lambda} \qquad \Lambda = \sqrt{a(a + 4\hat{\theta})} \qquad 27)$$

In the limit, the size of the coefficient on the lagged price level is bounded from below by zero and bounded from above by 1.

In short, as $\hat{\mu} \to \infty, \hat{\theta} \to 0, \dfrac{2\hat{\theta}}{a + 2\hat{\theta} + \Lambda} \to 0, p_t \to p^*$.

And as $\hat{\mu} \to 0, \hat{\theta} \to \infty, \dfrac{2\hat{\theta}}{a + 2\hat{\theta} + \Lambda} \to 1, p_t \to p_{t-1} + u_t$.

Thus the price level exhibits persistence for $0 < \hat{\mu} < \infty$.

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[11] The positive square root is appropriate as it implies $\hat{\theta} \to \infty$ as $\mu \to 0$ and $\hat{\theta} \to 0$ as $\mu \to \infty$.

Subtracting p_{t-1} from both sides of equation 27) yields the rate of inflation:

$$\pi_t = \frac{2ap^*}{a+\Lambda} + \frac{2\hat{\theta}u_t - (a+\Lambda)p_{t-1}}{a+2\hat{\theta}+\Lambda} \qquad 28)$$

Under a strict price level targeting scheme, $\hat{\mu} \to \infty,\ \hat{\theta} \to 0,\ \pi_t \to p^* - p_{t-1}$.

For the opposite extreme we have: $\hat{\mu} \to 0,\ \hat{\theta} \to \infty,\ \pi_t \to u_t$.

For the output gap the reduced form equation is:

$$y_t = \frac{(\Lambda - a)p^*}{(a+\Lambda)\hat{\theta}} - \frac{2(u_t + p_{t-1})}{2\hat{\theta}+a+\Lambda} \qquad 29)$$

Again applying the procedure laid out in the appendix yields the variances of the price level and the output gap under flexible price level targeting:

$$V(p_t^{PLT}) = \frac{(2\hat{\theta})^2\sigma_u^2}{(a+\Lambda)(a+\Lambda+4\hat{\theta})} \qquad 30)$$

$$V(y_t^{PLT}) = \frac{4\sigma_u^2}{(a+\Lambda)(a+\Lambda+4\hat{\theta})} \qquad 31)$$

As a final step, we derive the variance of the rate of inflation under the flexible price level targeting regime from equation 28):[12]

$$V(\pi_t^{PLT}) = \frac{2(2\hat{\theta})^2}{(a+\Lambda+2\hat{\theta})(a+\Lambda+4\hat{\theta})}\sigma_u^2 \qquad 32)$$

Table *2* provides summary information about the behavior of inflation and the output gap under a flexible price-level target for the case of autocorrelated cost-push shocks.

[12] See the appendix for further details on how this variance is calculated.

Table 2: Flexible Price-Level Targeting under Discretion: Persistence in the Cost-Push Disturbance.

$u_t = \rho u_{t-1} + \hat{u}_t \quad \hat{u}_t \sim N(0, \sigma_{\hat{u}}^2)$	
$V(\pi_t) = \dfrac{\beta^2}{1-\alpha\rho}\left[\dfrac{2(1-\rho)}{1+\alpha}\right]\sigma_u^2$	$\pi_t = \dfrac{2ap^*}{a+\Lambda} + \dfrac{2\hat{\theta}u_t - (\Lambda(1-\rho) + a(1+\rho))p_{t-1}}{a+2\hat{\theta}(1-\rho)+\Lambda}$
$V(y_t) = \left[\dfrac{\beta^2(1+\alpha\rho)}{\hat{\theta}^2(1-\alpha\rho)(1-\alpha^2)}\right]\sigma_u^2$	$y_t = \dfrac{(\Lambda-a)p^*}{(a+\Lambda)\hat{\theta}} - \dfrac{2\hat{\theta}u_t - (2\hat{\theta}(1-\rho)+\rho(\Lambda-a))p_{t-1}}{\hat{\theta}(2\hat{\theta}(1-\rho)+a+\Lambda)}$

$$\alpha = \frac{2\hat{\theta}(1-\rho)+\rho(\Lambda-a)}{a+2\hat{\theta}(1-\rho)+\Lambda} \quad \beta = \frac{2\hat{\theta}}{a+2\hat{\theta}(1-\rho)+\Lambda} \quad \Lambda = \sqrt{a(a+4\hat{\theta})} \quad \sigma_u^2 = \frac{1}{1-\rho^2}\sigma_{\hat{u}}^2$$

IV. A DISCRETIONARY SPEED LIMIT POLICY

In a recent contribution, Walsh (2003) puts forth the argument that the Federal Reserve has in practice pursued a speed limit policy. He arrives at this conclusion after examining statements made by members of the Board of Governors and studying press releases from the Federal Open Market Committee.[13] The distinguishing characteristic of a speed limit policy is that it focuses on the *change* in the output gap rather than the output gap proper. Naturally, the rate of inflation also remains a policy objective.

Under the speed limit policy the policymaker's objective is to minimize the expected loss function that consists of the weighted sum of the variance of the *change* in the output gap and the rate of inflation:

$$E[L_t] = V(y_t - y_{t-1}) + \mu^{SL}V(\pi_t) \qquad 33)$$

$V(\bullet)$ = the variance of the respective variable.

μ^{SL} = policymaker's aversion to inflation variability relative to output *growth* variability. $\mu^{SL} \geq 0$

[13] The term "speed limit" appears in a speech made by Governor Edward Gramlich in 1999: *Solving a standard model of the macroeconomy, such a policy would effectively convert monetary policy into what might be called "speed limit" form, where policy tries to ensure that aggregate demand grows at roughly the expected rate of increase of aggregate supply, which increase can be more easily predicted….(*Remarks, Wharton Public Policy Forum Series, Philadelphia, 1999 and reported by Walsh (2003))

The speed limit policy is set by an optimizing policymaker who acts with discretion. This policy involves choosing θ^{SL}, the weight on the *change* in the output gap, so that the objective function is minimized. We label this systematic relationship the speed limit policy rule. The rule appears in equation 34) below.

$$\theta^{SL}(y_t - y_{t-1}) + \pi_t = 0 \qquad 34)$$

To determine the behavior of the output gap and the rate of inflation under the speed limit policy, we combine the building blocks of the forward-looking model with the above policy rule. Substitute the Phillips Curve and the IS relation into equation 34) and solve the resulting expression for r_t. This expression is then substituted back into the IS relation to obtain:

$$y_t = \frac{\theta^{SL}}{\theta^{SL} + a} y_{t-1} - \frac{1}{\theta^{SL} + a}(E_t \pi_{t+1} + u_t) \qquad 35)$$

Combining equation 35) with equation 34) yields the reduced form equation for the rate of inflation:

$$\pi_t = \frac{a\theta^{SL}}{\theta^{SL} + a} y_{t-1} + \frac{\theta^{SL}}{\theta^{SL} + a}(E_t \pi_{t+1} + u_t) \qquad 36)$$

Just like policy from the timeless perspective and a price-level targeting strategy, the speed limit policy causes the current output gap and the current rate of inflation to depend on the past.

On the assumption that the current expectation of inflation next period is constant, the variances of the change of the output gap and the rate of inflation under a speed limit policy are:

$$V(y_t - y_{t-1}) = \left(\frac{1}{\theta^{SL} + a}\right)^2 \left[\frac{a}{2\theta^{SL} + a} + 1\right]\sigma_u^2 \qquad 37)$$

$$V(\pi_t) = \left(\frac{\theta^{SL}}{\theta^{SL} + a}\right)^2 \left[\frac{a}{2\theta^{SL} + a} + 1\right]\sigma_u^2 \qquad 38)$$

Substituting equations 37) and 38) into the policymaker's objective function, equation 33), and carrying out the minimization exercise yields the optimal policy parameter under the speed limit policy:

$$\theta^{SL} = \frac{2 - a^2 \mu^{SL} \pm \sqrt{4 + a^2 \mu^{SL} (5 + a^2 \mu^{SL})}}{3a\mu^{SL}} \qquad 39)$$

To calculate the expectation of the rate of inflation in period *t+1* that appears in equations 35) and 36), we apply again the minimum state variable approach. As the policy rule is based on the *growth* rate of the output gap, the putative solutions for the two endogenous variables contain the lagged output gap:

$$y_t = \phi_{11} u_t + \phi_{12} y_{t-1} \qquad 40)$$

$$\pi_t = \phi_{21} u_t + \phi_{22} y_{t-1} \qquad 41)$$

Under the speed limit policy, the solutions for the output gap and the rate of inflation are:

$$y_t = \frac{1}{\theta^{SL} + a + \phi_{22}} [\theta^{SL} y_{t-1} - (1 + \phi_{21} \rho) u_t] \qquad 42)$$

$$\pi_t = \frac{\theta^{SL}}{\theta^{SL} + a + \phi_{22}} [(a + \phi_{22}) y_{t-1} + (1 + \phi_{21} \rho) u_t] \qquad 43)$$

$$u_t = \rho u_{t-1} + \hat{u}_t \qquad \phi_{21} = \frac{\theta^{SL}}{\theta^{SL}(1 - \rho) + a + \phi_{22}}$$

$$0 \leq \rho < 1$$

$$\phi_{22} = \frac{-a \pm \sqrt{a^2 + 4a\theta^{SL}}}{2}$$

Before proceeding, we have to comment on the choice of roots for θ^{SL} and ϕ_{22}. In both cases, the positive root is the relevant root as it ensures that $\theta^{SL} \to \infty$ as $\mu^{SL} \to 0$ and $\theta^{SL} \to 0$ as $\mu^{SL} \to \infty$. Likewise the positive root guarantees that $\phi_{22} \to 0$ as $\theta^{SL} \to 0$ and that $\phi_{22} \to \infty$ as $\theta^{SL} \to \infty$.

The variances of the output gap and the rate of inflation under the speed limit policy appear in the table below.

Table 3: The Variances of the Output Gap and the Rate of Inflation under a Speed Limit Policy

$V(y_t) =$	$\dfrac{\beta^2(1+\alpha\rho)}{(1-\alpha^2)(1-\alpha\rho)}\sigma_u^2$
$V(\pi_t) =$	$\dfrac{2\theta^{SL^2}\beta^2(1-\rho)}{(1+\alpha)(1-\alpha\rho)}\sigma_u^2$

Note: $\sigma_u^2 = \dfrac{1}{1-\rho^2}\sigma_{\hat{u}}^2$

$$\alpha = \frac{\theta^{SL}}{\theta^{SL} + a + \phi_{22}} \qquad \beta = -\frac{(1+\phi_{21}\rho)}{\theta^{SL} + a + \phi_{22}}$$

V. THE DELEGATION ISSUE UNDER CONSTANT EXPECTATIONS

In this section we explore the delegation issue. The basic problem that society is confronted with can be briefly described as follows. Suppose society's loss function conforms to the standard loss function that includes the variances of the rate of inflation and the output gap. Suppose further that it is not possible to commit the policymaker, a central banker, to conduct policy from a timeless perspective. The central banker thus conducts monetary policy by discretion.

The objective of society is to achieve the outcomes for the rate of inflation and the output gap that obtain under optimal monetary policy from a timeless perspective. Is there a way for society to induce the policymaker to carry out policy in such a way so that the outcomes for the rate of inflation and real output correspond exactly to those under policy from a timeless perspective? In the remainder of this section, we show that society can realize this objective by delegating a price level target to the central banker or by instructing him to follow a speed limit policy. Critical in this respect is that society appoints a central banker with desirable preferences. Under price level targeting the central banker must have the appropriate degree of aversion to price level variability. Under a speed limit policy it is essential to find a central banker who puts the correct weight on the variance of the rate of inflation in the expected loss function that contains the variance of the *change* in the output gap as an argument.

Consider the policy rules associated with policy from a timeless perspective, price level targeting, and the speed limit policy, respectively:

$$\theta(y_t - y_{t-1}) + \pi_t = 0 \qquad \text{(Timeless Perspective)} \qquad 7)$$

$$\hat{\theta} y_t + p_t - p^* = 0 \qquad \text{(Price level target)} \qquad 17)$$

$$\theta^{SL} (y_t - y_{t-1}) + \pi_t = 0 \qquad \text{(Speed limit policy)} \qquad 34)$$

Here we see immediately that the only difference between the policy rule that governs the speed limit policy and policy from a timeless perspective concerns the size of the weight on the change in the output gap. After taking first differences of the policy rule associated with the price level target, we obtain:

$$\hat{\theta}(y_t - y_{t-1}) + \pi_t = 0 \qquad 44)$$

Again, the only difference between the dynamic form of the policy rule under a price level target and the rule under policy from a timeless perspective concerns the weight the policymaker attaches to the change in the output gap.[14] In Section III we established the optimal $\hat{\theta}$ under price level targeting:

$$\hat{\theta} = \frac{3 - a^2 \hat{\mu} + \sqrt{(-3 + a^2 \hat{\mu})^2 + (4a)^2 \hat{\mu}}}{4a\hat{\mu}} \qquad 24)$$

Under optimal policy from a timeless perspective the optimal policy parameter obeys the following relationship.

$$\theta = \frac{1}{\mu a} \qquad 45)$$

Next we set the optimal $\hat{\theta}$ associated with discretionary policymaking and a price level target equal to the optimal θ associated with policymaking from a timeless perspective and an inflation target.

[14] The policy rule under average inflation targeting cannot be reconciled with the policy rule that underlies policy under the timeless perspective. Let $\frac{\pi_t + \pi_{t-1}}{2} + \overline{\theta} y_t = 0$ represent the policy rule for two-period average inflation

$$\frac{3-a^2\hat{\mu}+\sqrt{(-3+a^2\hat{\mu})^2+(4a)^2\hat{\mu}}}{4a\hat{\mu}}=\frac{1}{\mu a} \qquad 46)$$

Solving for $\hat{\mu}$ yields

$$\hat{\mu}=\frac{\mu(3+2a^2\mu)}{2+a^2\mu} \qquad 47)$$

As $3+2a^2\mu > 2+a^2\mu$ it follows that $\hat{\mu}>\mu$. Society must appoint a *more conservative* central banker and delegate to him a price level target to ensure that the behavior of inflation and the output gap under discretion mimics the behavior of inflation and the output gap from a time-less perspective. This central banker is more conservative in the sense that his aversion to price level variability exceeds society's aversion to inflation variability.[15]

Under a speed limit policy, the policymaker can also set policy so as to mimic the behavior of the output gap and the rate of inflation that occurs under optimal policy from a timeless perspective. All that is required for society is to find the policymaker who has the requisite aversion to inflation variability. To determine the appropriate degree of aversion, we first set θ^{SL} equal to θ and then solve for μ^{SL}:

$$\theta^{SL}=\frac{2-a^2\mu^{SL}\pm\sqrt{4+a^2\mu^{SL}(5+a^2\mu^{SL})}}{3a\mu^{SL}}=\theta=\frac{1}{\mu a} \qquad 48)$$

Solving for μ^{SL} yields:

$$\mu^{SL}=\frac{\mu(4+3a^2\mu)}{3+2a^2\mu} \qquad 49)$$

---

targeting. This rule can be rewritten as $\frac{p_t-p_{t-2}}{2}+\bar{\theta}y_t=0$ which is not consistent with the rule under policy from a timeless perspective or the rule governing price level targeting. Hence the delegation issue becomes moot.

15 Strictly speaking, $\hat{\mu}$ ought to be scaled by $\frac{V(p_t)}{V(\pi_t)}$ so that the two relative weights can be directly compared. The scale factor can be obtained by dividing equation 30) by equation 32). Doing so yields $\hat{\mu}\frac{(a+\Lambda+2\hat{\theta})}{2(a+\Lambda)}$. For three different parameter values (*a=0.05, 0.25, 0.33;* $0<\mu<10$) the scaled relative weight in the policymaker's loss function exceeds the relative weight in society's loss function: $\hat{\mu}\frac{(a+\Lambda+2\hat{\theta})}{2(a+\Lambda)}>\mu$, thus warranting the appointment of a conservative central banker.

Comparing the numerator with the denominator, we observe that $4+3a^2\mu > 3+2a^2\mu$ which in turn implies that $\mu^{SL} > \mu$.[16]

Comparing the three weights, we find $\mu < \mu^{SL} < \hat{\mu}$. This finding confirms that much like price level targeting a speed limit policy also requires a conservative central banker to ensure that both the rate of inflation and the output gap mimic their behavior under optimal policy from a timeless perspective. However, in comparison, a speed limit policy requires a less conservative central banker than a price-level targeting strategy as $\mu^{SL} < \hat{\mu}.$ Figure 1 illustrates the relationship between $\mu, \hat{\mu},$ and μ^{SL} .[17]

VI. SUMMARY AND CONCLUSION

This paper has explored an issue that arises in the delegation process. The main concern of the paper is with the way the policymaker handles the forward-looking expectations of private agents. The treatment of these expectations in the policy setting stage turns out to be of critical importance for the attainment of society's basic objectives and for the type of central banker society wants to appoint. The paper shows that if expectations are treated as constant, then a *conservative* central banker can replicate the behavior of output and inflation under policy from a timeless perspective by engaging in price-level targeting or following a speed limit policy.

This result does not carry over to the case where the policymaker treats expectations as a dynamic process and takes account of the effect of changes in policy on expectations. In such a setting, a speed limit policy (Walsh, (2003)) cannot replicate the behavior of output and inflation produced by optimal policy at all while price level targeting (Vestin, (2000)) can do so only for white noise disturbances. Vestin's analysis suggests further that under price-level targeting the central banker is about as *conservative* as under inflation targeting.

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[16] Again, the relative weight in the policymaker's loss function ought to be multiplied by a scale factor. Under a speed limit policy the scale factor is $\frac{V(y_t)}{V(y_t - y_{t-1})} = \frac{1}{2(1-\theta^{SL}\eta)}$; $\eta = \frac{1}{\theta^{SL} + a + \phi_{22}}$. The choice of a conservative banker is called for if $\mu^{SL}\frac{1}{2(1-\theta^{SL}\eta)} > \mu$. This condition is satisfied by three different values of the key parameter a ($a=0.05, 0.25, 0.33;\ 0 < \mu < 10$). Vestin (2000) and Walsh (2003) apply scale factors as well.

[17] As both discretionary price level targeting and the discretionary speed limit policy achieve the optimal outcome for the rate of inflation and the output gap under policy from a timeless perspective, both policies generate a better trade-off between output and inflation variability than inflation targeting under discretion.

REFERENCES:

Dennis, Richard, “Optimal Policy in Rational Expectations Models: New Solution Algorithms,’ mimeo, Federal Reserve Bank of San Francisco, 2001.

Froyen, R. and Guender A. 2007, *Optimal Monetary Policy under Uncertainty*, Edward Elgar, Cheltenham.

Guender, A. 2006, “Price Level Targeting and the Delegation Issue”, manuscript, University of Canterbury.

Jensen, Henrik, “Targeting Nominal Income Growth or Inflation?” *American Economic Review*, 92, 4, September 2002: 928-956.

McCallum, Bennett, T, “On Non-Uniqueness in Rational Expectations Models: An Attempt at Perspective”, *Journal of Monetary Economics,* 11, March 1983: 139-168.

McCallum, Bennett T. and Edward Nelson, “Timeless Perspective vs. Discretionary Monetary Policy in Forward-Looking Models,” *Federal Reserve Bank of St. Louis Review,* 86, 2, March / April 2004: 43-56.

Nessén, Marianne, and David Vestin, “Average Inflation Targeting,” *Journal of Money, Credit, and Banking* 37, 5, October 2005: 837-863

Svensson, Lars, E.O., “Price-Level Targeting versus Inflation Targeting: A Free Lunch?” *Journal of Money, Credit, and Banking*, 31, 3, August 1999: 277-295.

Vestin, David, “Price-Level Targeting Versus Inflation Targeting in a Forward-Looking Model,” Working Paper 106, Swedish Riksbank, May 2000.

Walsh, Carl E., “The Output Gap and Optimal Monetary Policy,” *American Economic Review*, 93, 1, March 2003: 265-278.

Woodford, Michael, “Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?” in *New Challenges for Monetary Policy*: A Symposium Sponsored by the Federal Reserve Bank of Kansas City. Federal Reserve Bank of Kansas City, 1999: 277-316.

__________, “Optimal Policy Inertia”, *Manchester School* 67 (Supplement), 1999: 1-35.

APPENDIX:

Solution Procedure

To obtain the actual variance of the rate of the price level and the output gap, respectively, we proceed in the following way. Our point of departure is Equation 27) and Equation 29). Ignoring the constant terms, we write the two equations in matrix form:

$$\begin{bmatrix} y_t \\ p_t \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{2}{2\hat{\theta}+a+\Lambda} \\ 0 & \dfrac{2\hat{\theta}}{2\hat{\theta}+a+\Lambda} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} -\dfrac{2}{2\hat{\theta}+a+\Lambda} \\ \dfrac{2\hat{\theta}}{2\hat{\theta}+a+\Lambda} \end{bmatrix} \begin{bmatrix} u_t \end{bmatrix}$$

Rewrite the above in vector form:

$$\mathbf{X_t} = \mathbf{BX_{t-1}} + \mathbf{CU_t}$$

Forming the variance-covariance matrix yields

$$\mathbf{E[X_tX_t^{'}] = E[(BX_{t-1} + CU_t)(X_{t-1}^{'}B^{'}+U_t^{'}C')]}$$

$$\mathbf{E[X_tX_t^{'}] = BE[X_{t-1}X_{t-1}']B'+CE[U_tU_t^{'}]C'}$$

$$\mathbf{\Phi = B\Phi B^{'} + C\Omega C'}$$

Next, vectorizing by stacking columns yields

$$\mathbf{\Phi^v = (B\Phi B^{'} + C\Omega\, C')^v}$$

$$\mathbf{\Phi^v = (B\otimes B)\, \Phi^v + (C\otimes C)\ \Omega^v}$$

$$\mathbf{\Phi^v = (I - B\otimes B)^{-1} (C\otimes C)\Omega^v} \qquad \text{(A1)}$$

where

Φ is the variance-covariance matrix of $\mathbf{X_t}$

Ω is the variance-covariance matrix of $\mathbf{U_t}$.

The construction of **B⊗B** proceeds as follows. Define **B** as

$$\mathbf{B} = \begin{bmatrix} 0 & -\dfrac{2}{2\hat{\theta}+a+\Lambda} \\ 0 & \dfrac{2\hat{\theta}}{2\hat{\theta}+a+\Lambda} \end{bmatrix}$$

Then **B⊗B**= $\begin{bmatrix} 0 & 0 & 0 & \frac{4}{(2\hat{\theta}+a+\Lambda)^2} \\ 0 & 0 & 0 & -\frac{4\hat{\theta}}{(2\hat{\theta}+a+\Lambda)^2} \\ 0 & 0 & 0 & -\frac{4\hat{\theta}}{(2\hat{\theta}+a+\Lambda)^2} \\ 0 & 0 & 0 & \frac{4\hat{\theta}^2}{(2\hat{\theta}+a+\Lambda)^2} \end{bmatrix}$ is a 4x4 matrix

It follows then that

(I - B⊗B)= $\begin{bmatrix} 1 & 0 & 0 & -\frac{4}{(2\hat{\theta}+a+\Lambda)^2} \\ 0 & 1 & 0 & \frac{4\hat{\theta}}{(2\hat{\theta}+a+\Lambda)^2} \\ 0 & 0 & 1 & \frac{4\hat{\theta}}{(2\hat{\theta}+a+\Lambda)^2} \\ 0 & 0 & 0 & 1-\frac{4\hat{\theta}^2}{(2\hat{\theta}+a+\Lambda)^2} \end{bmatrix}$

Proceeding in similar fashion, we obtain **C⊗C**= $\begin{bmatrix} \frac{4}{(2\hat{\theta}+a+\Lambda)^2} \\ -\frac{4\hat{\theta}}{(2\hat{\theta}+a+\Lambda)^2} \\ -\frac{4\hat{\theta}}{(2\hat{\theta}+a+\Lambda)^2} \\ \frac{4\hat{\theta}^2}{(2\hat{\theta}+a+\Lambda)^2} \end{bmatrix}$

Notice that $\mathbf{\Phi^v}$= $\begin{bmatrix} V(y_t) \\ Cov(y_t, p_t) \\ Cov(p_t, y_t) \\ V(p_t) \end{bmatrix}$ and $\mathbf{\Omega^v}$= $\begin{bmatrix} V(u_t) \\ \end{bmatrix}$

The model assumes that the cost-push disturbance follows a white noise process: $V(u_t) = \sigma_u^2$.

After taking the inverse of **(I - B⊗B),** we can employ Equation (A1) obtain the variances of real output and the price level. They take the following form:

$$\begin{bmatrix} V(y_t) \\ V(p_t) \end{bmatrix} = \begin{bmatrix} \dfrac{4}{(a+\Lambda)(a+\Lambda+4\hat{\theta})} \\ \dfrac{4\hat{\theta}^2}{(a+\Lambda)(a+\Lambda+4\hat{\theta})} \end{bmatrix} V(u_t) \qquad \text{(A2)}$$

To obtain the variance of inflation, we proceed as follows. Consider first Equation 28) in the text:

$$\pi_t = \frac{2ap^*}{a+\Lambda} + \frac{2\hat{\theta}u_t - (a+\Lambda)p_{t-1}}{a+2\hat{\theta}+\Lambda} \qquad \text{(A3)}$$

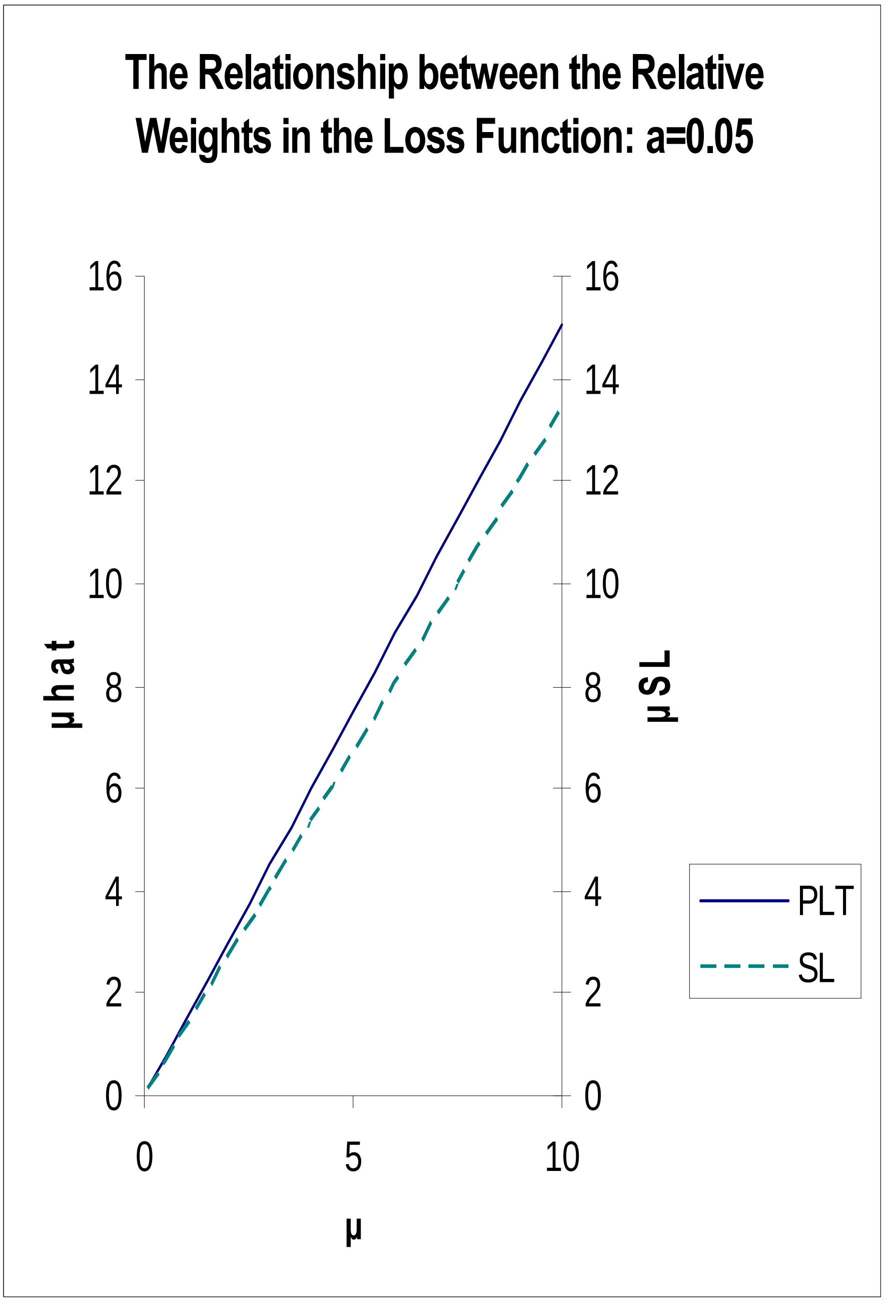
Calculating the variance of inflation and letting $V(p_t) = \sigma_p^2$ yields:

$$V(\pi_t) = \frac{(2\hat{\theta})^2\sigma_u^2}{(a+2\hat{\theta}+\Lambda)^2} + \frac{(a+\Lambda)^2\sigma_p^2}{(a+2\hat{\theta}+\Lambda)^2} \qquad \text{(A4)}$$

Substituting the variance of the price level from Equation (A2) into (A4) and manipulating the resulting expression yields the variance of inflation stated in the text:

$$V(\pi_t^{PLT}) = \frac{2(2\hat{\theta})^2}{(a+\Lambda+2\hat{\theta})(a+\Lambda+4\hat{\theta})}\sigma_u^2 \qquad \text{(A5)}$$

Figure 1:


The Relationship between the Relative Weights in the Loss Function: a=0.05
16
14
12
10
8
6
4
2
0
μ hat
μ SL
0
5
10
μ
PLT
SL